FDTD Analysis of Magnetized Ferrites: An Approach Based on the Rotated Richtmyer Difference Scheme

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***Abstract*— Electromagnetic wave propagation in magnetized ferrites is modelled by solving Maxwell's time-dependent curl equations coupled with the equation of motion of the magnetization vector. A discretization approach based on the rotated Richtmyer finite-difference scheme is proposed. The new approach has been used to calculate the phase constants of transversally magnetized ferrite-loaded waveguides. The numerical dispersion equation for TE$_{n0}$ modes is derived. The results obtained with this approach for a ferrite-filled and a ferrite-slab loaded waveguide are compared with those obtained with Yee's scheme extended for the treatment of ferrites and with the exact results.**

I. Introduction

The finite-difference time-domain (FDTD) method, as formulated by Yee [1], is now a well-established numerical technique for the analysis of a great variety of electromagnetic problems. It is based on the direct discretization of Maxwell's time-dependent curl equations by using central differences. In order to approximate the spatial derivatives by central differences, a single field component is assigned to each node of the unit cell.

Very recently, Yee's formulation of the FDTD method has been extended to include magnetized ferrites [2]. In this extension, the treatment of ferrite material is based on discretizing not only Maxwell's equations but also the equation of motion of the magnetization vector. Since Yee's formulation involves the assignment of a single field component to each mesh node, the equation of motion, which only involves time derivatives, requires spatial interpolation to be evaluated.

This letter presents an alternative approach for handling ferrite material. This approach is also based on the discretization of Maxwell's equations coupled with the equation of motion of the magnetization vector but instead of using an extended Yee scheme, it makes use of the rotated Richtmyer finite-difference scheme. From the point of view of the ferrite treatment, this latter scheme, which has recently been proposed as a way of solving Maxwell's equations for the isotropic case [3], has the important advantage that the discretization mesh has only two different kinds of nodes, electric nodes and magnetic nodes. As a consequence, the equation of motion can be evaluated without any type of spatial interpolation.

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II. Approach

It is assumed that the ferrite is magnetized to saturation by a dc magnetic field applied in the z-direction, $\vec{H}_i = H_i\vec{a}_z$. Under this condition, the ferrite is described by the linearized equation of motion expressed in terms of the magnetic flux density B and the magnetic field strength H (for the sake of brevity no magnetic losses are included) [4]

$$\frac{\partial B_x}{\partial t} - \mu_o\frac{\partial H_x}{\partial t} = -\gamma\mu_0(H_iB_y - (M_s + H_i)\mu_0H_y) \quad (1)$$

$$\frac{\partial B_y}{\partial t} - \mu_o\frac{\partial H_y}{\partial t} = -\gamma\mu_0((M_s + H_i)\mu_0H_x - H_iB_x), \quad (2)$$

where γ is the gyromagnetic ratio, M_s the saturation magnetization and μ_0 the permeability of the vacuum.

These equations coupled with Maxwell's equations must be solved. As has recently been proposed for isotropic media [3], Maxwell's equations are discretized by using the rotated Richtmyer scheme. The isotropic formulation uses a mesh with electric nodes (where E_x, E_y, and E_z are defined) and magnetic nodes (where H_x, H_y, and H_z are defined). To extend this mesh to the ferrite case, B_x and B_y are added at the magnetic nodes. The discretization in time of (1)–(2) follows the same scheme as in [2]. This mesh arrangement allows the time-difference version of these equations to be evaluated without spatial interpolation. Note that, as in the isotropic case, spatial interpolation is used to evaluate Maxwell's curl equation.

III. Test of Accuracy: Comparison with Yee's Scheme

In order to show the validity of the above scheme and compare its accuracy with the Yee scheme, the new approach for treating ferrite materials has been applied to the analysis of transversally magnetized ferrite-loaded waveguides. In general, this is a 3-D problem that can be reduced to an equivalent 2-D problem by assuming that the fields have the form $F(x,y,z,t) = f(x,z,t)\exp(-j\beta y)$ [5], where y is the direction of propagation and β the phase constant of the mode being considered. To obtain the dispersion characteristics $\beta(f)$, β is fixed as an input parameter, and the time-domain response is obtained by applying the reduced 2-D FDTD formulation in the guide's cross-section, which has previously been discretized using a 2-D mesh (see Fig. 1). The frequency-domain response is obtained from the spectral analysis of the

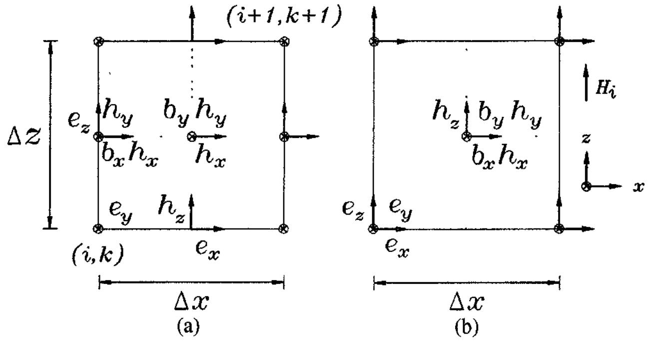

Fig. 1. 2-D mesh for the analysis of ferrite-loaded waveguides. (a) Yee mesh extended to ferrite treatment. (b) Rotated Richtmyer mesh extended to ferrite treatment.

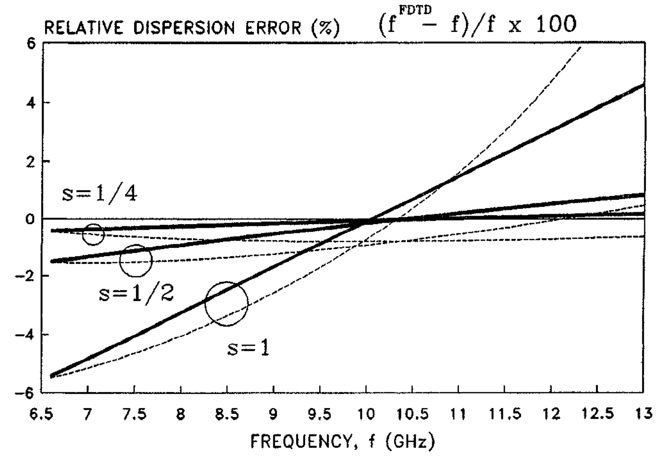

Fig. 2. Relative error in the determination of frequencies of the TE_{10} mode of a ferrite-filled rectangular waveguide magnetized in the z-direction as a function of frequency for various values of the stability factor s. $a = 22.86$ mm, $\epsilon_r = 9$, $4\pi M_s = 2000$ G, $H_i = 200$ Oe, and $\Delta x = a/10$. Solid line: Yee scheme extended to ferrite treatment. Dashed line: Rotated Richtmyer scheme extended to ferrite treatment.

time-domain response. Each peak of the spectrum corresponds to one excited mode that has the previously mentioned value of β at the frequency of the peak. By changing β and repeating this process, the whole dispersion diagram is obtained.

To illustrate the accuracy of the method, we have derived the dispersion equation for TE_{n0} modes (E_z, H_x, H_y, B_x, and B_y components only) of a transversally magnetized lossless ferrite-filled waveguide. As was done in the isotropic case [6], we substitute general plane wave solutions into the difference equations to obtain the numerical dispersion relation

$$\frac{\epsilon\mu_{\text{eff}}^{\text{FDTD}}}{(\Delta t)^2}\sin^2(\omega\Delta t/2) = \frac{\sin^2(k_x\Delta x/2)}{(\Delta x)^2} + \frac{\beta^2}{4}, \quad (3)$$

for Yee's scheme, where $\mu_{\text{eff}}^{\text{FDTD}}$ is the numerical effective permeability given by

$$\mu_{\text{eff}}^{\text{FDTD}} = \mu_0 \frac{4\tan^2(\omega\Delta t/2) - (\gamma\mu_0(H_i + M_s)\Delta t)^2}{4\tan^2(\omega\Delta t/2) - (\gamma\mu_0\Delta t)^2 H_i(M_s + H_i)}. \quad (4)$$

Δx is the space increment in the x-direction, Δt the time step, ω the numerical angular frequency and k_x the numerical wavenumber in the x-direction. The dispersion equation for the Richtmyer scheme is obtained by the same procedure. It has the same form as (3) but the term $(\beta^2/4)$ is replaced by $(\beta\cos(k_x\Delta x/2)/2)^2$.

Consider the TE_{10} mode of a rectangular ferrite-filled waveguide of width a. Assuming a wavenumber $k_x = \pi/a$ and selecting a value of β, we can obtain the corresponding numerical value f^{FDTD} by solving the numerical dispersion equation analytically. The dispersion error is then obtained by comparing f^{FDTD} with the exact value f. The accuracy of both formulations, depends on Δx and Δt, which are related by the stability condition. For 1-D problems, like the problem under consideration, the stability condition for the Yee scheme can be expressed as [6]

$$s = \frac{c}{\sqrt{\epsilon_{r_{\min}}}}\Delta t\left(\frac{1}{(\Delta x)^2} + \frac{\beta^2}{4}\right)^{\frac{1}{2}}, \quad (5)$$

while for the Richtmyer scheme, it becomes [7]

$$s = \frac{c\Delta t}{\sqrt{\epsilon_{r_{\min}}}\Delta x}, \quad (6)$$

where s is the stability factor, whose value must not exceed unity to guarantee the numerical stability of the algorithms.

The relative dispersion error $((f^{\text{FDTD}} - f)/fx100)$ in the determination of the frequencies (corresponding to values of β in the band 6.5–13.0 GHz) of the mode TE_{10} of a rectangular ferrite-filled waveguide is shown in Fig. 2. In this example, $k_x\Delta x$ is constant, hence, the spatial dispersion error does not depend on the frequency. At frequencies near the cutoff, most of the error is incurred in the evaluation of $\mu_{\text{eff}}^{\text{FDTD}}$. This error becomes smaller as Δt decreases, i.e., as s decreases. At high frequencies, the error becomes larger because $\omega\Delta t$ increases; this time dispersion error also becomes smaller as s decreases.

Fig. 3 shows the phase constant for the TE_{10} mode of a ferrite-slab loaded waveguide. The results depicted in the figure have been obtained numerically with both schemes and are compared with the exact ones, which have been obtained by solving the corresponding characteristic equation [4]. In this case, $\epsilon_{r_{\min}} = 1$, hence, the error in $\mu_{\text{eff}}^{\text{FDTD}}$ is maintained very low. At high frequencies, the Richtmyer scheme exhibits a greater spatial dispersion error than Yee's scheme. Near the cutoff, similar accuracy is obtained with both schemes, but the Richtmyer scheme is more efficient (larger values of Δt, less CPU time and memory requirements).

IV. Conclusion

An alternative discretization scheme, based on the rotated Richtmyer finite- difference scheme, is proposed for the FDTD treatment of magnetized ferrites. The main advantage of this approach over the extended Yee scheme for ferrite treatment is that all the field components involved in the equation of motion are located at the same node of the cell, thus avoiding the use of spatial interpolations to evaluate this equation. However, as in the isotropic case, spatial interpolation is necessary to evaluate Maxwell's equations. Then, the rotated Richtmyer scheme presents greater accuracy in the treatment of the constitutive characteristics of the ferrite, but also higher numerical dispersion error than Yee's scheme. To compare

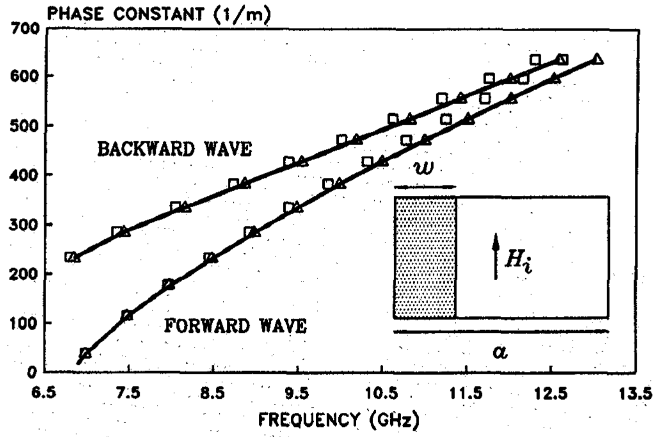

Fig. 3. Phase constant of the TE_{10} mode of a rectangular waveguide loaded with a ferrite slab. $a = 22.86$ mm, $a/w = 3$, $\epsilon_r = 9$, $4\pi M_s = 2000$ G, $H_i = 200$ Oe, $\Delta x = a/12$ and $s = 1/2$. Solid line: exact. Triangles: Yee scheme extended to ferrite treatment. Squares: Rotated Richtmyer scheme extended to ferrite treatment.

these two schemes, they have been applied to the analysis of ferrite-loaded waveguides. From the results obtained, it seems that the Richtmyer scheme would be better under high anisotropic conditions and the Yee scheme advantageous at high frequencies. Further discussion on the relationship between these two difference schemes can be found in [8].

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